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CORRECTION TO "THE FINITE-DIMENSIONAL P_λ SPACES WITH SMALL λ"

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The purpose of this note is to give a proof of Lemma 3 of our paper [1]. The proof given in [1] is incorrect. We wish to thank S. J. Bernau for pointing out the error in the proof to us. The statement of Lemma 3 of [1] is the following:

Let Y be a Banach space, let X be a subspace of Y and let T be a projection of Y onto X with $\| T \| \leqq 1 + \gamma$, *where* $\gamma < 1/20$. *Let P be a projection on Y with* $\| P \| \leqq 1 + \gamma$ *such that* $\| TPx - Px \| \leqq \gamma \| x \|$ *for all x in X. Then there is a projection Q on X such that* $\| Q \| < 1 + 20\gamma$ *and* $\| Qx - Px \| \leqq 20\gamma \| x \|$ *for all x in X.*

It clearly suffices to prove the existence of a projection Q on X with $\| Q \| = 1 + 20\gamma$ and $\| Qx - TPx \| \leqq 19\gamma \| x \|$ for all x in X. The operator $U = \| TP_{|X} \|$ satisfies the inequality

$$\| U^2 - U \| = \| (TPTP - TP)_{|X} \| \leqq \| TP \| \; \| (TP - P)_{|X} \| \leqq (1 + \gamma)^2 \gamma.$$

Hence, in order to prove the above statement it will be enough to prove the following

PROPOSITION. *Let U be an operator on a Banach space X such that* $\| U \| = N$ *and* $\| U^2 - U \| \leqq c$ *where* $c < 1/8$. *Then there is a projection Q on X such that* $\| Q - U \| \leqq (2N + 1) g \operatorname{Exp}(2g)$ *where* $g = 2c(1 - 4c)^{-1}$.

PROOF OF THE PROPOSITION. Let $S = 2U - I$ where I denotes the identity on X. Then $\| S^2 - I \| = \| 4U^2 - 4U \| \leqq 4c$ and $\| S \| \leqq 2N + 1$. Put $4c = d$ and

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$2N+1=M$. Clearly S^2 is invertible, $(S^2)^{-1}=\sum_{n=0}^{\infty}(I-S^2)^n$ and $\|(S^2)^{-1}\|\leqq(1-d)^{-1}$. Hence $S^{-1}=S(S^2)^{-1}$ and

$$\|S^{-1}\|=(1-d)^{-1}\|S\|\leqq(1-d)^{-1}M.$$

Now construct by induction a sequence of operators (S_n) on X such that $S_1=S$ and for each $n\geqq 2$, $S_n=\frac{1}{2}(S_{n-1}^{-1}+S_{n-1})$. Clearly $\|S_n\|\leqq\frac{1}{2}(\|S_{n-1}^{-1}\|+\|S_{n-1}\|)$ and

$$\|I-S_n^2\|=\tfrac{1}{4}\|(S_{n-1}^{-1}-S_{n-1})^2\|=\tfrac{1}{4}\|S_{n-1}^{-2}(I-S_{n-1}^2)^2\|$$

$$\leqq\tfrac{1}{4}\|S_{n-1}^{-2}\|\,\|I-S_{n-1}^2\|^2\leqq\tfrac{1}{4}(1-\|I-S_{n-1}^2\|)^{-1}\|I-S_{n-1}^2\|^2.$$

Put $d_n=\|I-S_n^2\|$ and $M_n=\|S_n\|$, then the above estimates imply that $d_n\leqq\frac{1}{4}(1-d_{n-1})^{-1}d_{n-1}^2$ and, because $0\leqq d_n\leqq\frac{1}{2}$ for all n, $d_n\leqq\frac{1}{2}d_{n-1}^2$ and

$$M_n\leqq M_{n-1}(1-d_{n-1})^{-1}(1-\tfrac{1}{2}d_{n-1})\leqq M_{n-1}(1+d_{n-1}).$$

It follows that for each $n\geqq 1$, $d_n\leqq\frac{1}{2}d^n$ and

$$M_n\leqq M\prod_{i=1}^{n-1}(1+d^i)\leqq M\,\mathrm{Exp}((1-d)^{-1}d)=L.$$

Moreover

$$\|S_{n+1}-S_n\|=\tfrac{1}{2}\|S_n^{-1}-S_n\|\leqq\tfrac{1}{2}\|S_n^{-1}\|\,\|I-S_n^2\|=\tfrac{1}{2}M_n(1-d_n)^{-1}d_n\leqq Ld^n.$$

Hence (S_n) converges to an involution J where

$$\|J-S\|=\left\|\sum_{n=1}^{\infty}(S_{n+1}-S_n)\right\|\leqq L(1-d)^{-1}d=M(1-d)^{-1}d\,\mathrm{Exp}((1-d)^{-1}d).$$

Put $Q=\frac{1}{2}(I+J)$, then Q is the desired projection. This proves the Proposition.

REFERENCE

1. M. Zippin, *The finite-dimensional P_λ spaces with small λ*, Isr. J. Math. **39** (1981), 359–364.

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